

3.6 WS → #'s 7+8

Finding Bounds on Zeros

Let F denote a polynomial function whose leading coefficient is 1.

$$F(x) = x^n + a_{n-1}x^{n-1} + \dots + a_1x + a_0$$

A bound M on the zeros of f is the smaller of the following 2 values:

- ① $\text{MAX} \{ 1, |a_0| + |a_1| + \dots + |a_{n-1}| \}$
- or
- ② $1 + \text{MAX} \{ |a_0|, |a_1|, \dots, |a_{n-1}| \}$

where $\text{MAX} \{ \}$ means "choose the largest entry in $\{ \}$."

7.) Find a bound to the zeros of each polynomial, then graph.

A.) $F(x) = x^5 + 3x^3 - 9x^2 + 5$ B.) $g(x) = 4x^5 - 2x^3 + 2x^2 + 1$

* The leading coefficient of F is already 1, so we don't have to factor it out. * factor out a 4

① $\text{MAX} \{ 1, |3| + |-9| + |5| \} = 17$ ① $\text{MAX} \{ 1, |-\frac{1}{2}| + |\frac{1}{2}| + |\frac{1}{4}| \} = \frac{5}{4}$

② $1 + \text{MAX} \{ |3|, |-9|, |5| \} = 1+9 = 10$ ② $1 + \text{MAX} \{ |-\frac{1}{2}|, |\frac{1}{2}|, |\frac{1}{4}| \} = 1 + \frac{1}{2} = \frac{3}{2}$

* The smaller of the two #'s is ⑩, * The smaller of the two #'s

so 10 is the bound. Thus, the zeros of F must fall b/w -10 + 10. circles above is $\frac{5}{4}$, so the bound is $\frac{5}{4}$. Zeros of g are b/w $-\frac{5}{4} + \frac{5}{4}$

7 (continued) \rightarrow For graphing part of #7, just graph on your graphing CALCs (using the bounds) AS your X MIN + X MAX.

8.) Find the real zeros of the polynomial function
 $f(x) = x^5 - 1.8x^4 - 17.79x^3 + 31.672x^2 + 37.95x - 8.7121$

- Quickly find the bounds to determine your X-MIN + X-MAX:

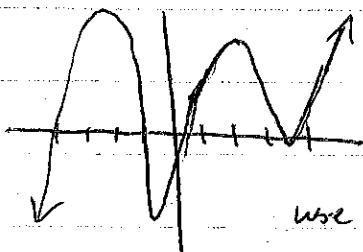
① $\text{MAX} \{ 1, |-1.8| + |-17.79| + |31.672| + |37.95| + |-8.7121| \}$

$\hookrightarrow = \text{MAX} \{ 1, 97.9241 \} = 97.9241$

② $1 + \text{MAX} \{ |-1.8|, |-17.79|, |31.672|, |37.95|, |-8.7121| \}$

$1 + 37.95 = 38.95$

The bound is 38.95, so $X\text{-min} = -38.95$ + $X\text{-max} = 38.95$ to start. You'll see after graphing that you could narrow the $X\text{-min}$ + $X\text{max}$ to -5 + 5 to get a good looking graph.



Zeros at $\rightarrow x = -4, -1, 2, \dots$

use the table around 3.3 to see that there is a zero b/w 3.29 + 3.30, AND another zero b/w 3.30 + 3.31.